NNLO electroweak corrections for polarized Møller scattering: one-loop insertions to boxes

A. G. Aleksejevs[*]

Memorial University of Newfoundland, Corner Brook, Canada

S. G. Barkanova[†]

Acadia University, Wolfville, Canada

Yu. M. Bystritskiy[‡]

Joint Institute for Nuclear Research, Dubna, Russia

E. A. Kuraev

Joint Institute for Nuclear Research, Dubna, Russia

V. A. Zykunov[§]

Belarusian State University of Transport, Gomel, Belarus

April 15, 2015

[*]e-mail: aaleksejevs@grenfell.mun.ca

[†]e-mail: svetlana.barkanova@acadiau.ca

[‡]e-mail: bystr@theor.jinr.ru

[§]e-mail: vladimir.zykunov@cern.ch

1

Abstract

The paper discusses the two-loop (NNLO) electroweak radiative corrections to the parity-violating Møller scattering asymmetry induced by insertions to boxes of electron and neutrino mass operators (fermion self-energies), vertex functions and boson self-energies. The results will be relevant to the ultra-precise 11 GeV MOLLER experiment planned at the Jefferson Laboratory, which will measure the weak charge of the electron and search for new physics. The numerical estimations for the NNLO contribution to the cross section asymmetry are presented.

PACS: 12.15.Lk, 12.20.Ds, 13.40.Em.

1 Introduction

The Møller scattering [1] with polarized electrons has attracted active interest from both experimental and theoretical standpoints for several reasons. It has allowed the high-precision determination of the electron-beam polarization at SLC [2], SLAC [3, 4], JLab [5] and MIT-Bates [6] (and as a future prospect — the ILC [7]). The polarized Møller scattering can be an excellent tool in measuring parity-violating weak interaction asymmetries [8]. The first observation of Parity Violation (PV) in the Møller scattering was made by the E-158 experiment at SLAC [9, 10, 11], which studied scattering of 45- to 48-GeV polarized electrons on the unpolarized electrons of a hydrogen target. It results at $Q^2 = -t = 0.026 \text{ GeV}^2$ for the observable parity-violating asymmetry $A_{PV} = (1.31 \pm 0.14 \text{ (stat.)} \pm 0.10 \text{ (syst.)}) \times 10^{-7}$ [12] which allowed

one of the most important parameters in the Standard Model (SM) – the sine of the Weinberg angle $\sin\theta_W$ – to be determined with accuracy of $0.5\,\%$

The MOLLER (Measurement Of a Lepton Lepton Electroweak Reaction) experiment planned at the Jefferson Lab aims to measure the parity-violating asymmetry in the scattering of 11 GeV longitudinally-polarized electrons from the atomic electrons in a liquid hydrogen target with a combined statistical and systematic uncertainty of $2\,\%$ [14, 15, 16, 17]. At such precision, any inconsistency with the Standard Model (SM) predictions will clearly signal the new physics. However, a comprehensive analysis of radiative corrections is needed before any conclusions can be made. Since MOLLER's stated precision goal is significantly more ambitious than that of its predecessor E-158, theoretical input for this measurement must include not only a full treatment of one-loop (next-to-leading order, NLO) electroweak radiative corrections but also two-loop corrections (next-to-next-leading order, NNLO).

The significant theoretical effort has been dedicated to one-loop radiative corrections already. A short review of the references on that topic is done in [18, 19], where we calculated a full set of the one-loop electroweak corrections (EWC) both numerically with no simplifications using computer algebra packages and by-hand in a compact form analytically free from non-physical parameters, and found the total relative correction to the observable asymmetry to be close to -70%. It is possible that a large theoretical uncertainty in the prediction for the asymmetry may come from two-loop corrections. One way to find some indication of the size of higher-order contributions is to compare results that are expressed in terms of quantities

related to different renormalization schemes. In [20], we provided a tuned comparison between the result obtained with different renormalization conditions, first within one scheme then between two schemes. Our calculations in the on-shell and Constrained Differential Renormalization schemes show the difference of about 11%, which is comparable with the difference of 10% between $\overline{\text{MS}}$ [21] and the on-shell scheme [22]. It is also worth noting that although two-loop corrections to the cross section may seem to be small, it is much harder to estimate their scale and behavior for such a complicated observable as the parity-violating asymmetry to be measured by the MOLLER experiment.

The two-loop EWC to the Born cross section ($\sim \mathcal{M}_0 \mathcal{M}_0^+$) can be divided onto two classes: Q-part induced by quadratic one-loop amplitudes $\sim \mathcal{M}_1 \mathcal{M}_1^+$, and T-part – the interference of Born and two-loop amplitudes $\sim 2\,\text{Re}\left(\mathcal{M}_0 \mathcal{M}_2^+\right)$ (here index i in the amplitude $\mathcal{M}_i$ corresponds to the order of perturbation theory). The Q-part was calculated exactly in [23] (using Feynman–t'Hooft gauge and the on-shell renormalization), where we show that the Q-part is much higher than the planned experimental uncertainty of MOLLER, i.e. the two-loop EWC are larger than was assumed in the past. The large size of the Q-part demands detailed and consistent treatment of T-part, but this formidable task will require several stages. Our first step was to calculate the gauge-invariant double boxes [24]. In this paper we do the next step – we consider the EWC arising from the contribution of a wide class of the gauge-invariant Feynman amplitudes of the box type with one-loop insertions: fermion mass operators [or Fermion Self-Energies in Boxes (FSEB)], vertex functions [or Vertices in Boxes (VB)], and polarization of

4

vacuum for bosons [or Boson Self-Energies in Boxes (BSEB)].

The paper is organized as follows. We define the basic notations in Sect. 2 and present FSEB, VB, and BSEB in Sect. 3. In Sect. 4, we provide the numerical results for asymmetry for the kinematics conditions of the MOLLER experiment and discuss work still to be done in the future. In Appendix A, the mass operators of electron and neutrino are presented. In Appendix B, we show the result for one-loop corrections to vertex functions for the case when only one fermion is on the mass shell. In Appendix C, we consider the polarization of vacuum for the virtual photon, Z- and W-boson. The details of calculation of ultraviolet cut-off loop momenta integrals can be found in Appendix D.

2   Basic notations

We consider the process of electron-electron elastic scattering, i.e. Møller process:

$$e_-(p_1, \lambda_1) + e_-(p_2, \lambda_2) \to e_-(p_3, \lambda_3) + e_-(p_4, \lambda_4), \tag{1}$$

where λ_i $(i = \overline{1,4})$ are the chiral states of initial and final electrons. The kinematical invariants were defined in the standard way:

$$s = (p_1 + p_2)^2, \qquad t = (p_1 - p_3)^2, \qquad u = (p_1 - p_4)^2. \tag{2}$$

In the MOLLER experiment, the expected beam energy is $E_{\text{beam}} = 11$ GeV, that is $s = 2mE_{\text{beam}} \approx 0.01124$ GeV^2, where m is the electron mass ($p_i^2 = m^2$). For the central region of MOLLER (at $\theta \sim 90°$ in center-of-mass system of initial electrons), $-t \approx -u \approx s/2$ thus we can use an approximation

that s, $|t|$, $|u| \gg m^2$. Also, as for MOLLER kinematics in central region s, $|t|$, $|u| \ll m_{Z,W}^2$ we neglect in following the terms of order $\mathcal{O}(s/m_{Z,W})$.

We consider the process (1) in terms of chiral amplitudes $\mathcal{M}^\lambda$, where $\lambda = \{\lambda_1\lambda_2\lambda_3\lambda_4\}$ is the chiral state of initial and final electrons. The PV asymmetry to be measured by MOLLER is then defined as

$$A = \frac{|\mathcal{M}^{----}|^2 - |\mathcal{M}^{++++}|^2}{\sum_\lambda |\mathcal{M}^\lambda|^2}, \qquad \sum_\lambda |\mathcal{M}^\lambda|^2 = 2(8\pi\alpha)^2 \frac{s^4 + t^4 + u^4}{t^2 u^2}. \quad (3)$$

In the Born approximation, this asymmetry has a form

$$A^{(0)} = \frac{s}{2m_W^2} \frac{s^2 t u}{s^4 + t^4 + u^4} \frac{a}{s_W^2} \quad (4)$$

proportional to

$$a = 1 - 4s_W^2. \quad (5)$$

Let us now recall that s_W (c_W) is the sine (cosine) of the Weinberg angle expressed in terms of the Z- and W-boson masses according to the Standard Model rules:

$$s_W = \sqrt{1 - c_W^2}, \qquad c_W = m_W/m_Z. \quad (6)$$

Thus, the factor a is just $a \approx 0.109$ and the asymmetry is therefore suppressed by both s/m_W^2 and a. Even at at $\theta = 90°$, where the Born asymmetry is maximal, it is extremely small:

$$A^{(0)} = \frac{s}{9m_W^2} \frac{a}{s_W^2} \approx 9.4968 \cdot 10^{-8}. \quad (7)$$

We denote the specific contribution to the asymmetry by the index C, which thus can be BSEB, FSEB, VB or IB=BSEB+FSEB+VB for the whole set of diagrams Fig. 1, respectively.

6

The contribution to the asymmetry $(\Delta A)_C$ and the relative correction D_A^C are defined as:

$$(\Delta A)_C = \frac{|\mathcal{M}_C^{----}|^2 - |\mathcal{M}_C^{++++}|^2}{\sum |\mathcal{M}_0^\lambda|^2}, \tag{8}$$

$$D_A^C = \frac{(\Delta A)_C}{A^{(0)}} = \frac{|\mathcal{M}_C^{----}|^2 - |\mathcal{M}_C^{++++}|^2}{|\mathcal{M}_0^{----}|^2 - |\mathcal{M}_0^{++++}|^2}. \tag{9}$$

The relative correction **to observable asymmetry** from the contribution of type C looks as (see derivation in more details in [19]):

$$\delta_A^C = \frac{A^C - A^{(0)}}{A^{(0)}} = \frac{D_A^C - \delta^C}{1 + \delta^C}, \tag{10}$$

where the relative correction to unpolarized cross section σ_{00}^0 (we used short notation for differential cross section $\sigma \equiv d\sigma/d(\cos\theta)$) is:

$$\delta^C = \frac{\sigma_{00}^C}{\sigma_{00}^0}. \tag{11}$$

For the two-loop effects where δ^C is small, we can use an approximate equation for relative correction to asymmetry $\delta_A^C \approx D_A^C$.

3 Insertion of mass operator, vertex and vacuum polarization functions to the box type amplitude

The numerical value of loop momentum squared $|k^2|$ in the box-type amplitudes with the heavy boson exchange is large compared with the square of electron mass $|k^2| \gg m^2$, since if $|k^2|$ is far from $M_{Z,W}^2$ the contribution is suppressed with the mass of heavy boson squared in denominator. So we can
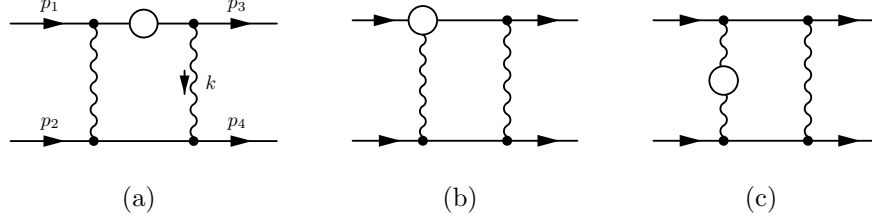
Figure 1: (a) Fermion self energies in boxes (FSEB), (b) boxes with vertices (VB), (c) boson self energies in boxes (BSEB). In this diagrams all wavy lines are assumed to be photons or Z-bosons. We also considered crossed box diagrams which can also contain W-bosons legs.

use the asymptotic expressions for the one-loop vertex functions as well as the mass and vacuum polarization operators. Using the well-known approach [25], [26] which we successfully employed for the box-type chiral amplitudes in [24] (see also [27]), we can write for the direct ZZ-box chiral amplitude of "$++++$" type:

$$\bar{u}_3\gamma_\mu(a+\gamma_5)\hat{k}\gamma_\nu(a+\gamma_5)u_1 \ \bar{u}_4\gamma^\mu(a+\gamma_5)(-\hat{k})\gamma^\nu(a+\gamma_5)u_2 =$$
$$= -\frac{(a+1)^4}{gf}\mathrm{Sp}\left[\hat{p}_3\gamma_\mu\hat{k}\gamma_\nu\hat{p}_1\hat{p}_2\hat{p}_4\gamma^\mu\hat{k}\gamma^\nu\hat{p}_2\hat{p}_1\omega_+\right] = -\frac{8k^2s^2t(a+1)^4}{gf}. \quad (12)$$

Easily we can get a similar expression for the crossed box and for amplitude of "$----$" type. The quantities g and f in (12) coincide with a and b from [24], respectively, and are defined as:

$$g = \bar{u}_1\omega_-\hat{p}_2\omega_+u_4, \qquad f = \bar{u}_2\omega_-\hat{p}_1\omega_+u_3,$$

where $\omega_\pm = (1\pm\gamma_5)/2$ are the chirality projection operators. Let us calculate t-channel amplitude; the u-channel amplitude can be obtained by the replacement $t\leftrightarrow u$. This interchange will be denoted below as an operator P_{tu}.

8

The box-type amplitude with the double Z-boson exchange with all the possible insertions (i.e. VB, FSEB and BSEB) has a form:

$$\mathcal{M}^{ZZ,\pm} = i\frac{\alpha^2(1\pm a)^4}{(4c_W s_W)^4}\frac{6s^2 t}{gf m_Z^2}\int\limits_0^\infty\frac{d\tau}{(1+\tau)^2}I_{ZZ}(\tau), \tag{13}$$

where "$\pm$" sign corresponds to the chiral amplitudes $\mathcal{M}^{\pm\pm\pm\pm}$. The expression for the box amplitude with $Z\gamma$-exchange is similar:

$$\mathcal{M}^{Z\gamma,\pm} = i\frac{2\alpha^2(1\pm a)^2}{(4c_W s_W)^2}\frac{6s^2 t}{gf m_Z^2}\int\limits_0^\infty\frac{d\tau}{\tau(1+\tau)}I_{Z\gamma}(\tau). \tag{14}$$

At last, for $\gamma\gamma$-exchange amplitude we have:

$$\mathcal{M}^{\gamma\gamma,\pm} = i\alpha^2\frac{6s^2 t}{gf m_Z^2}\int\limits_z^\infty\frac{d\tau}{\tau^2}I_{\gamma\gamma}(\tau). \tag{15}$$

In all the above cases, the integration variable is related to the loop momentum as $\tau = -k^2/m_Z^2$. The lower limit of integration $z = -t/m_Z^2$ for $\mathcal{M}^{\gamma\gamma}$ is introduced to avoid the double counting for the region of small loop momenta squares $-k^2 < s$, where we use the Yennie–Frautchi–Suura approach [28]. Finally, the contribution to $\mathcal{M}^{----}$ arises from the box-type Feynman diagram with two W-boson exchange:

$$\mathcal{M}_{WW}^{----} = -i\frac{\alpha^2}{2s_W^4}\frac{s^2 t}{gf m_W^2}\int\limits_0^\infty\frac{d\tau_W}{(1+\tau_W)^2}I_{WW}(\tau_W), \qquad \tau_W = -k^2/m_W^2. \tag{16}$$

The structure of the quantities I_{ij} in (13), (14), (15) and (16) corresponds

to three types of radiative corrections, FSEB, VB and BSEB, respectively:

$$I_{ZZ} = 2\mathcal{M}_e + 4V_{eeZ} + 2\Pi_{ZZ},$$

$$I_{Z\gamma} = 2\mathcal{M}_e + 2V_{eeZ} + 2V_{ee\gamma} + \Pi_{ZZ} + \Pi_{\gamma\gamma}, \qquad (17)$$

$$I_{\gamma\gamma} = 2\mathcal{M}_e + 4V_{ee\gamma} + 2\Pi_{\gamma\gamma},$$

$$I_{WW} = 2\mathcal{M}_\nu + 4V_{e\nu W} + 2\Pi_{WW}.$$

Here, we use the dimensionless quantities for the product of fermion Green function and the truncated mass operators of electron M_e and neutrino M_ν (see Appendix A):

$$\mathcal{M}_{e,\nu} = \frac{i\hat{k}}{k^2} M_{e,\nu}. \qquad (18)$$

The vertex function $V_{ee\gamma}^\mu(k^2)$ with one electron on the mass shell and another electron off the mass shell is normalized as

$$V_{ee\gamma}^\mu(k^2) = -ie\gamma^\mu \; V_{ee\gamma}(k^2), \qquad V_{ee\gamma}(0) = 0. \qquad (19)$$

The vertex function $V_{eeZ}^\mu(k^2)$ is normalized at the point $k^2 = m_Z^2$:

$$V_{eeZ}^\mu(k^2) = -\frac{ie}{4c_W s_W}\gamma^\mu \; V_{eeZ}(k^2), \qquad V_{eeZ}(m_Z^2) = 0, \qquad (20)$$

and similarly for $e\nu W$-vertex function we have:

$$V_{e\nu W}^\mu(k^2) = \frac{ie}{\sqrt{2}s_W}\gamma^\mu\omega_- V_{e\nu W}(k^2), \qquad V_{e\nu W}(m_W^2) = 0. \qquad (21)$$

The explicit expressions for the vertices $V_{ee\gamma}$, V_{eeZ} and $V_{e\nu W}$ are given in Appendix B.

The dimensionless products of boson Green function with the relevant regularized polarization operator $\Pi_{\mu\nu}(q) = \Pi(q^2)g_{\mu\nu} + B(q^2)q_\mu q_\nu$ are defined

10

as:

$$\Pi_\gamma = \frac{-i}{q^2}\Pi_{\gamma\gamma}^{\mathrm{tr}}(q^2), \qquad\qquad \Pi_{\gamma\gamma}^{\mathrm{tr}}(0) = \frac{\partial}{\partial q^2}\Pi_{\gamma\gamma}^{\mathrm{tr}}(0) = 0;$$

$$\Pi_Z = \frac{-i}{q^2 - m_Z^2}\Pi_{ZZ}^{\mathrm{tr}}(q^2), \qquad \Pi_{ZZ}^{\mathrm{tr}}(m_Z^2) = \frac{\partial}{\partial q^2}\Pi_{ZZ}^{\mathrm{tr}}(m_Z^2) = 0;$$

$$\Pi_{Z\gamma} = \frac{-i}{q^2}\Pi_{Z\gamma}^{\mathrm{tr}}(q^2), \qquad\qquad \Pi_{Z\gamma}^{\mathrm{tr}}(0) = 0;$$

$$\Pi_W = \frac{-i}{q^2 - m_W^2}\Pi_{WW}^{\mathrm{tr}}(q^2), \qquad \Pi_{WW}^{\mathrm{tr}}(m_W^2) = \frac{\partial}{\partial q^2}\Pi_{WW}^{\mathrm{tr}}(m_W^2) = 0. \quad (22)$$

The structure $B(q^2)q_\mu q_\nu$ does not contribute due the gauge invariance. The explicit expression for the "truncated" quantities are given in Appendix C.

4   Numerical results and conclusion

For the numerical calculations, we use the central kinematical point of the MOLLER experiment and α, m_W and m_Z in accordance with the Particle Data Group [29]. The effective quark masses used for the vector boson self-energy loop contributions are extracted from the shifts in the fine structure constant due to hadronic vacuum polarization $\Delta\alpha_{\mathrm{had}}^{(5)}(m_Z^2) = 0.02757$ [30]. For the mass of Higgs boson, we take $m_H = 125$ GeV.

The contribution relevant to the observed asymmetry is the interference of the two-loop box-type amplitudes with the Born amplitudes $\mathcal{M}_{\gamma,Z}$. The contribution to the matrix element squared (i.e. cross section) has the form:

$$|\mathcal{M}_{\mathrm{IB}}^{\pm\pm\pm\pm}|^2 = 2\left(1 + P_{tu}\right)\left[\left(\mathcal{M}^{ZZ} + \mathcal{M}^{Z\gamma} + \mathcal{M}^{WW}\right)\mathcal{M}_\gamma^* + \mathcal{M}^{\gamma\gamma}\mathcal{M}_Z^*\right]. \quad (23)$$

In the right-hand side of this equation, we assume that the amplitudes are taken in the same chiral state corresponding to the state of left-hand

side. Note that the intermediate states with $W^{\pm}$ bosons and Faddeev–Popov ghosts $G_W^{\pm}$ contribute to the mass and vertex operators in the $\mathcal{M}^{----}$ chiral amplitude. Since the parameter a is very small, we can present the final result as:

$$\left|\mathcal{M}_{\text{IB}}^{----}\right|^2 - \left|\mathcal{M}_{\text{IB}}^{++++}\right|^2 = -H(a) + (H(-a) + Y) = -2a\frac{\partial H(a)}{\partial a}\Big|_{a\to 0} + Y,$$

(24)

and thus the relative correction D_A^{IB} has the form:

$$D_A^{\text{IB}} = \frac{t^2 u^2}{128\,(\pi\alpha)^2\,(s^4 + t^4 + u^4)}\left(-2a\frac{\partial H(a)}{\partial a}\Big|_{a\to 0} + Y\right)\frac{1}{A^{(0)}}.$$

(25)

We define H and Y as:

$$H = H_{ZZ} + H_{Z\gamma} + H_{\gamma\gamma} + H_{WW} + H_{\text{mix}},$$

(26)

$$Y = Y_{ZZ} + Y_{Z\gamma} + Y_{\gamma\gamma} + Y_{WW} + Y_{\text{mix}},$$

where the first four terms in both H and Y correspond to the box-type amplitudes with ZZ, $Z\gamma$, $\gamma\gamma$ and WW bosons exchanged between electrons, and the last term corresponds to the cases with Z or γ and the mixed boson Green function with polarization operator $\Pi_{Z\gamma}$.

Using the following relations (see, for example, [24] and [27])

$$\frac{1}{gf}\left(\frac{1}{gf} - \frac{1}{cd}\right)^* = -\frac{1}{st^2u}; \qquad \frac{1}{gf}\left(\frac{t}{gf} - \frac{u}{cd}\right)^* = \frac{2}{s^2t}$$

(27)

12

we obtain the following numerical results:

$$H_{ZZ} = -\frac{3\alpha^3\pi(1+a)^4}{8(c_W s_W)^4}\frac{s^3}{m_Z^2 tu}(1+P_{tu})\int\limits_0^\infty \frac{d\tau}{(1+\tau)^2}\left[2(M_e^\gamma + (1+a)^2 M_e^Z)+\right.$$

$$\left. +2\Pi_{ZZ} + 4(V^\gamma + (1+a)^2 V^Z)\right] =$$

$$= -1.653\cdot 10^{-13}(1+a)^4\left(-81.36 - 1.1293(1+a)^2\right)$$

$$Y_{ZZ} = -\frac{3\alpha^3\pi}{8(c_W s_W)^4}\frac{s^3}{m_Z^2 tu}(1+P_{tu})\int\limits_0^\infty \frac{d\tau}{(1+\tau)^2}[2M_e^W + 4(V^\nu + V^{2\nu})] =$$

$$= 3.139\cdot 10^{-12};$$

$$H_{Z\gamma} = -\frac{12\alpha^3\pi(1+a)^2}{(c_W s_W)^2}\frac{s^3}{m_Z^2 tu}(1+P_{tu})\int\limits_0^\infty \frac{d\tau}{\tau(1+\tau)}[2(M_e^\gamma + (1+a)^2 M_e^Z)+$$

$$+ \Pi_{ZZ} + \Pi_{\gamma\gamma} + 2(V^\gamma + V_{ee\gamma}^\gamma + (1+a)^2(V^Z + V_{ee\gamma}^Z))] =$$

$$= -9.155\cdot 10^{-11}(1+a)^2\left(-4.30744 - 0.04567(1+a)^2\right);$$

$$Y_{Z\gamma} = -\frac{12\alpha^3\pi}{(c_W s_W)^2}\frac{s^3}{m_Z^2 tu}(1+P_{tu})\int\limits_0^\infty \frac{d\tau}{\tau(1+\tau)}[2M_e^W + 2(V^\nu + V^{2\nu} + V_{ee\gamma}^W)] =$$

$$= 6.974\cdot 10^{-11},$$

$$H_{\gamma\gamma} = \frac{12\alpha^3\pi(1+a)^2}{(c_W s_W)^2}(1+P_{tu})\frac{s^2 z}{m_Z^2 t}\int\limits_z^\infty \frac{d\tau}{\tau^2}[2(M_e^\gamma + (1+a)^2 M_e^Z)+$$

$$+ 2\Pi_{\gamma\gamma} + 4(V_{ee\gamma}^\gamma + (1+a)^2 V_{ee\gamma}^Z)] =$$

$$= -3.094 \cdot 10^{-12}(1+a)^2(-2.52038 - 5.04456 \cdot 10^{-6}(1+a)^2),$$

$$Y_{\gamma\gamma} = \frac{12\alpha^3\pi}{(c_W s_W)^2}(1+P_{tu})\frac{s^2 z}{m_Z^2 t}\int\limits_z^\infty \frac{d\tau}{\tau^2}[2M_e^W + 4V_{ee\gamma}^W] = -4.4261 \cdot 10^{-17},$$

$$H_{WW} = 0;$$

$$Y_{WW} = \frac{8\alpha^3\pi}{(s_W)^4}\frac{s^3}{m_Z^2 tu}(1+P_{tu})\int\limits_0^\infty \frac{d\tau_W}{(1+\tau_W)^2}[2M_\nu + 2\Pi_{WW} + 4V_{eW\nu}] =$$

$$= -3.36 \cdot 10^{-10}. \tag{28}$$

The "mixed"-type amplitude in two-loop approximation has two different contributions $(H, Y)_{\mathrm{mix}} = (H, Y)_{\mathrm{mix}}^{(1)} + (H, Y)_{\mathrm{mix}}^{(2)}$. The first contribution is associated with the two-loop box-type amplitude:

$$H_{\mathrm{mix}}^{(1)} = -\frac{6\alpha^3\pi(1+a)}{(c_W s_W)^3}\frac{s^3}{m_Z^2 tu}(1+P_{tu})\int\limits_0^\infty \frac{d\tau_W}{(1+\tau_W)}R_{\gamma Z}(\tau_W)\times$$

$$\times\left((4c_W s_W)^2\frac{1}{\tau_W} + (1+a)^2\frac{1}{1+\tau_W}\right) =$$

$$= -1.10029 \cdot 10^{-9}(1+a)(0.007746 - 0.000340(1+a)^2),$$

$$Y_{\mathrm{mix}}^{(1)} = 0,$$

$$R_{\gamma Z}(\tau_W) = \frac{\alpha c_W}{8\pi s_W}\left(-3 + 2(3 - 2c_W^2)\frac{1}{\tau_W}\right)L_W(\tau_W),$$

$$L_W(\tau_W) = \int\limits_0^1 dx \log(1 + x(1-x)\tau_W).$$

The second contribution arises from the interference of the Born-type am-

14

plitude with the mixed Green function and the box type one-loop amplitude with the $\gamma\gamma$-exchange:

$$H_{\text{mix}}^{(2)} = \frac{48\alpha^3\pi(1+a)}{c_W s_W}(1+P_{tu})\frac{s^3 z}{t^2 u}R_{\gamma Z}(z) = -3.982 \cdot 10^{-13}(1+a),$$

$$Y_{\text{mix}}^{(2)} = 0. \tag{29}$$

The contributions to the asymmetry from the transition polarization operator $\Pi_{Z\gamma}$ with leptons in the fermion loop are proportional to higher powers of a, which is small. The same reasoning is valid for the quark-antiquark state contribution. Specifically, it enters with the factor

$$(2/3)(1 - (8/3)s_W^2) - (1/3)(1 - (4/3)s_W^2) = a/3. \tag{30}$$

The contributions from (W^+W^-), $(W^{\pm}G_W^{\mp})$, $(G_W^{\pm}G_W^{\mp})$ intermediate states are considered in Appendix C.

Finally, we are ready to present final numerical value for the relative corrections considered in this paper to the observable cross section asymmetry. The one-loop (NLO) corrections [18, 19] give the biggest contribution,

$$\delta_A^{\text{NLO}} = -0.6953. \tag{31}$$

Several categories of the NNLO contributions (Q-part and double boxes) are calculated in [23] and [24] and give the following values:

$$\delta_A^{\text{NLO}+Q} = -0.6535, \qquad \delta_A^{\text{double box}} \approx D_A^{\text{double box}} = -0.0101. \tag{32}$$

Summing up all the contributions in (25), the numerical result of the class of the gauge-invariant Feynman amplitudes considered in this paper (boxes

with one-loop insertions of fermion mass operators, vertex functions and polarization of vacuum for bosons) is:

$$\delta_A^{\text{IB}} \approx D_A^{\text{IB}} = -0.0039. \tag{33}$$

As one can see, the relative correction we obtained is much less than the expected MOLLER experimental error, but it still a non-negligible contribution to the MOLLER error budget. Most likely, the entire set of two-loop corrections will be smaller than the experimental statistical error, but, in the light of the MOLLER success depending so crucially on its precision, the two-loop corrections still need to be controlled.

As the low-energy precision experiment, MOLLER is complementary to the LHC efforts and may discover new physics signal that could escape LHC detection. However, for the MOLLER experiment to produce meaningful physics, the uncertainties in the NNLO EWC must be much smaller than the MOLLER statistical error. Clearly, there is a need for the a complete study of the two-loop electroweak radiative corrections in order to meet the MOLLER precision goals.

5   Acknowledgements

A Mass operators

Here, we will define the explicit form of the quantities M_e, M_ν which enter to I_{ij} from (17). The quantity M_e has the following form:

$$M_e = M_e^\gamma + (a \pm \gamma_5)^2 M_e^Z + \omega_-^2 M_e^W. \tag{34}$$

The explicit expression for the truncated mass operator in QED was found by R. Karplus and N. Kroll in 1950 [31, 32]:

$$M_e^\gamma = \frac{i\alpha}{2\pi m}(\hat{p} - m)^2 \left[\frac{1}{2(1-\rho)}\left(1 - \frac{2-3\rho}{1-\rho}\log\rho\right) - \right.$$
$$\left. - \frac{\hat{p}+m}{m\rho}\left(\frac{1}{2(1-\rho)}\left(2 - \rho + \frac{\rho^2 + 4\rho - 4}{1-\rho}\log\rho\right) + 1 + 2\log\frac{\lambda}{m}\right)\right], \tag{35}$$

$$\rho = 1 - \frac{p^2}{m^2}.$$

It is useful to note that the expression in the square brackets is finite at $\rho \to 1$. In the limit of large $\tau_1 = -p^2/m^2$ with logarithmical accuracy we have

$$M_e^\gamma = M_e^\gamma(\tau_1)\cdot\hat{p} \approx -\frac{\alpha}{4\pi}\log(\tau_1)\cdot\hat{p}, \qquad \tau_1 \gg 1. \tag{36}$$

This mass operator contribution to the integral in (15) with logarithmical accuracy gives:

$$-\frac{t}{m_Z^2}\int\limits_{-t/m_Z^2}^{\infty}\frac{d\tau}{\tau^2}M_e^\gamma(\tau_1) = -\frac{\alpha}{4\pi}\log\frac{-t}{m^2}. \tag{37}$$

The mass operators induced by additional Z and W bosons have the following form:

$$M_e^Z = \frac{\alpha}{2\pi(4c_W s_W)^2} \int\limits_0^1 (1-x)\log(1+\tau x)dx,$$

$$M_\nu = M_e^W = \frac{\alpha}{\pi s_W^2} \int\limits_0^1 (1-x)\log(1+\tau x)dx. \tag{38}$$

B Vertices

The general form of the vertex function is $V^\mu(k) = A\gamma^\mu + Bk^\mu$; the term Bk^μ inserted in the box-type amplitude gives no contribution due to the gauge invariance. The vertex function with one electron on the mass shell and other electron off the mass shell $V_{ee\gamma}^\mu(p, p-k, k) = -ie\gamma^\mu V_{ee\gamma}(k^2)$, normalized as $V_{ee\gamma}(0) = 0$, has three contributions:

$$V_{ee\gamma} = V_{ee\gamma}^\gamma + (a \pm \gamma_5)^2 V_{ee\gamma}^Z + \omega_-^2 V_{ee\gamma}^W. \tag{39}$$

First, let us consider the QED-type contribution with the virtual photon intermediate state $V_{ee\gamma}^\gamma$. The standard procedure of joining denominators and performing the loop momenta integration leads to

$$V_{ee\gamma}^\gamma(k^2) = \frac{\alpha}{4\pi} \int\limits_0^1 dx \int\limits_0^1 ydy \left(\log\frac{\Lambda^2}{D} + \frac{k^2 b\bar{b}}{2D}\right), \qquad b = xy, \qquad \bar{b} = 1-b,$$

$$\tag{40}$$

$$D = (m^2 - k^2 x(1-x))y^2 + (1-y)\lambda^2 - y(1-y)(k^2 - 2p_1 k),$$

where Λ is cut-off regularization parameter. Since the sub-set of the diagrams considered here is gauge invariant on its own, it was not essential for us to

18

use the dimensional regularization scheme providing gauge invariance, so we simply applied the cut-off technique. There is no significant numerical difference between two schemes in this situation.

The renormalization procedure consists in subtraction at $k = 0$ and leads to:

$$V_{ee\gamma}^{\gamma}(\tau_e) = -\frac{\alpha}{4\pi} \int\limits_0^1 \log(1 + x(1-x)\tau_e)dx, \qquad \tau_e = -\frac{k^2}{m^2}. \tag{41}$$

The contribution of this vertex function to the integral in (15) has the form:

$$-\frac{t}{m_Z^2} \int\limits_{-t/m_Z^2}^{\infty} \frac{d\tau}{\tau^2} V_{ee\gamma}^{\gamma}(\tau_e) \approx \frac{\alpha}{4\pi}\left(1 - \log\frac{-t}{m^2}\right). \tag{42}$$

The other contributions are:

$$V_{ee\gamma}^Z = \frac{\alpha}{2\pi(4c_W s_W)^2} \int\limits_0^1 dx \int\limits_0^1 y\, dy \left(\log\frac{1-y}{1-y+b\bar{b}\tau} - \frac{b\bar{b}\tau}{2(1-y+b\bar{b}\tau)}\right),$$

$$V_{ee\gamma}^W = \frac{\alpha}{4\pi s_W^2} \int\limits_0^1 dx \int\limits_0^1 y\, dy \left(3\log\frac{yc_W^2 + \tau b\bar{b}}{yc_W^2} - \frac{\tau b(\bar{b}-b)}{2(yc_W^2 + \tau b\bar{b})}\right). \tag{43}$$

Vertex function $V_{eeZ}^\mu = -iG\gamma^\mu(a\pm\gamma_5)V_{eeZ}$, $G = e/(4s_W c_W)$ has four different contributions:

$$V_{eeZ} = \omega_- V^\gamma + (a \pm \gamma_5)^2 V^Z + \omega_- V^\nu + V^{2\nu}, \tag{44}$$

19

and is normalized as $V_{eeZ}(k^2 = m_Z^2) = 0$. These contributions are

$$V^\gamma = -\frac{\alpha}{4\pi} \log \tau,$$

$$V^Z = \frac{1}{(4c_W s_W)^2} \frac{\alpha}{2\pi} \int_0^1 dx \int_0^1 ydy \times$$

$$\times \left(\log \frac{1 - y - b\bar{b}}{1 - y + b\bar{b}\tau} - \frac{b\bar{b}\tau}{2(1 - y + b\bar{b}\tau)} - \frac{b\bar{b}}{2(1 - y - b\bar{b})} \right), \qquad (45)$$

$$V^\nu = -\frac{\alpha c_W^2}{4\pi} \int_0^1 dx \int_0^1 ydy \times$$

$$\times \left(3 \log \frac{yc_W^2 + \tau b\bar{b}}{yc_W^2 - b\bar{b}} - \frac{\tau b(\bar{b} - b)}{2(yc_W^2 + \tau b\bar{b})} - \frac{b(\bar{b} - b)}{2(yc_W^2 - b\bar{b})} \right),$$

$$V^{2\nu} = \frac{\alpha}{2\pi s_W^2} \int_0^1 dx \int_0^1 ydy \times$$

$$\times \left(\log \frac{(1 - y)c_W^2 - b\bar{b}}{(1 - y)c_W^2 + \tau b\bar{b}} - \frac{\tau b\bar{b}}{2(yc_W^2 + \tau b\bar{b})} - \frac{b\bar{b}}{2(yc_W^2 - b\bar{b})} \right).$$

And finally, the vertex function $V^\mu_{e\nu W} = i\frac{\gamma_\mu \omega_-}{\sqrt{2}} V_{e\nu W}$ as well contains three contributions:

$$V_{e\nu W} = V^{ZW} + V^{WZ} + V^{\gamma W}, \qquad (46)$$

and is normalized as $V_{e\nu W}(\tau = -c_W^2) = 0$ and $V^{ZW} = V^{WZ}$. So the contribu-

20

tions are:

$$V^{ZW} = \frac{\alpha}{4\pi s_W^2} \int\limits_0^1 dx \int\limits_0^1 y\,dy \times$$

$$\times \left(-3\log\frac{ya_x + \tau b\bar{b}}{ya_x - c_W^2 b\bar{b}} + + \frac{\tau b(\bar{b} - b)}{2(ya_x + \tau b\bar{b})} + \frac{c_W^2 b(\bar{b} - b)}{2(ya_x - c_W^2 b\bar{b})} \right),$$

$$V^{\gamma W} = -\frac{\alpha}{4\pi} \left[\int\limits_0^1 dx \int\limits_0^1 dy \times \right.$$

$$\left. \times \left(3\log\frac{bc_W^2 + \tau + c_W^2}{bc_W^2} + \frac{\tau(\bar{b} - b)}{2(c_W^2 + \tau b)} \right) - 1 + \frac{1}{4}\log\frac{m_W^2}{m^2} + \frac{1}{4}\log\frac{m^2}{\lambda^2} \right],$$

where $a_x = x + (1 - x)c_W^2$. Note that the term containing $\log(m^2/\lambda^2)$ in expression $V^{\gamma W}$ can be omitted as it will be absorbed by the similar terms in two-loop contributions after applying the Yennie–Frautschi–Suura regularization (see [33] for details).

C   Polarization operators

While considering the vacuum polarization operators of photon, Z- and W-boson at one loop, one should recall that the regularization implies the double subtraction procedure. The "truncated" operators imply including only the vertices of interaction of bosons with the fermion loop. From now on, we will omit index "tr". The general form of the polarization operator is:

$$\Pi_{\mu\nu}(q) = g_{\mu\nu}\Pi(q^2) + q_\mu q_\nu B(q^2). \tag{47}$$

We only need to consider a part of polarization tensor proportional to $g_{\mu\nu}$. The reason is the gauge invariance of the whole set of the double-box amplitudes, which leads to a zero contribution for terms proportional to $q_\mu q_\nu$ tensor.

21

Let's define Π^γ as:

$$\Pi^\gamma = -\frac{i}{q^2}\Pi^\gamma(q^2). \tag{48}$$

It has five types of contributions, corresponding to the intermediate state of lepton–antilepton pairs, quark–antiquark pairs, W^+W^- and the charged ghost state $G_W^+ G_W^-$:

$$\Pi^\gamma = \Pi^l + \Pi^q + \Pi^{WW} + \Pi^{G_W^\pm G_W^\mp} + \Pi^{W^\pm G_W^\mp}. \tag{49}$$

The contribution of leptons and quarks are associated with the quadratic divergent integral over the loop momentum:

$$\frac{1}{4}\int \frac{dk}{(k^2 - m^2)\left((k-q)^2 - m^2\right)} \mathrm{Sp}\left[(\hat{k}+m)\gamma_\mu(\hat{k}-\hat{q}+m)\gamma_\nu\right]. \tag{50}$$

Using the set of divergent integrals (see Appendix D) and performing the regularization procedure, we include the contribution of leptons and quarks as

$$\Pi^l + \Pi^q = \frac{\alpha}{\pi\tau}\left(\sum_{l=e,\mu,\tau} G(\tau, \sigma^l) + 3\sum_{q=u,d,s,\cdots} Q_q^2 G(\tau, \sigma^q)\right), \tag{51}$$

where

$$G(\tau, \sigma) = \frac{1}{3}(\tau - 2\sigma)L(\frac{\tau}{\sigma}) + \frac{1}{9}\tau, \qquad L(z) = \int\limits_0^1 dx \log(1 + x(1-x)z),$$

$$\sigma^f = \frac{m_f^2}{m_Z^2}, \qquad \tau = -\frac{q^2}{m_Z^2},$$

The factor 3 takes into account the number of quark colours. The last three contributions in (49) are

$$\Pi^{WW} + \Pi^{G_W^\pm G_W^\mp} + \Pi^{W^\pm G_W^\mp} = -\frac{\alpha}{12\pi\tau}\left(\frac{1}{6}\tau + (5\tau - c_W^2)L\left(\frac{\tau}{c_W^2}\right)\right), \tag{52}$$

the known result for Feynman–t'Hooft gauge used in [34, 35].

The polarization operator for Z-boson has seven types of contributions:

$$\Pi^Z = \Pi^l_Z + \Pi^q_Z + \Pi^\nu_Z + \Pi^{W^+W^-}_Z + \Pi^{G^+_W G^-_W}_Z + \Pi^{G_1 G_2}_Z + \Pi^{W^\pm G^\mp_W}_Z, \qquad (53)$$

were we used the definition

$$\Pi^Z = -\frac{i}{q^2 - m_Z^2} \Pi^Z_{\text{tr}}(q^2). \qquad (54)$$

The contribution of lepton Π^l_Z, quark Π^q_Z and the neutrino Π^ν_Z loops can be calculated in the non-renormalized approach:

$$\Pi_{\mu\nu} = \frac{\alpha}{12\pi} \left(q^2 \log \frac{\Lambda^2}{q^2} + O(q^2) \right) g_{\mu\nu}. \qquad (55)$$

The renormalization of $R(\tau)$ for any contribution to the polarization operator of Z-boson consist of the following replacement:

$$R(\tau) \to R(\tau) - R(-1) - (\tau + 1)R'(-1). \qquad (56)$$

In particular, for example:

$$-q^2 \log \frac{q^2}{m^2} \to m_Z^2 F(\tau), \qquad F(\tau) = \tau \log \tau - 1 - \tau. \qquad (57)$$

Keeping in mind that there are three generations of charged leptons, neutrinos, and quarks, we obtain:

$$\Pi^{l+q+\nu}_Z = \frac{\alpha}{12\pi} \frac{F(\tau)}{1+\tau} \left[3 + \frac{3}{4(s_W c_W)^2} + \frac{1}{2(s_W c_W)^2} \left(1 - 2s_W^2 + \frac{20}{9} s_W^4 \right) \right]. \qquad (58)$$

The contribution of W^+W^- pair in the intermediate state to the Z-boson polarization operator looks like:

$$\Pi_Z^{W^+W^-} = \frac{\alpha c_W^2}{8\pi s_W^2} \frac{1}{1+\tau} \times$$
$$\times \left[\left(\frac{19}{6}\tau - \frac{16}{3} \right) L \left(\frac{\tau}{c_W^2} \right) - \left(\frac{19}{6}\tau - \frac{16}{3} \right) c_1 + \frac{17}{2}(\tau+1)c_2 \right],$$

$$c_1 = L \left(-\frac{1}{c_W^2} \right) \approx -0.248, \qquad c_2 = \int_0^1 \frac{x(1-x)}{1-x(1-x)/c_W^2} dx \approx 0.226.$$

The contribution of the charged ghosts $G_W^\pm$ is:

$$\Pi_Z^{G_W^+ G_W^-} = -\frac{\alpha(1-2s_W^2)^2}{4\pi(c_W s_W)^2} \frac{1}{1+\tau} \times$$
$$\times \left[\left(\frac{1}{12}\tau + \frac{1}{3}c_W^2 \right) \left(L \left(\frac{\tau}{c_W^2} \right) - c_1 \right) + \frac{1}{c_W^2}(\tau+1) \left(\frac{1}{12} - \frac{1}{3}c_W^2 \right) c_2 \right],$$
$$\Pi_Z^{W^\pm G_W^\mp} = -\frac{\alpha s_W^4}{2\pi} \frac{1}{1+\tau} \left[L \left(\frac{\tau}{c_W^2} \right) - c_1 - \frac{1}{c_W^2}c_2(1+\tau) \right].$$

And, finally, the contribution from the state with ghosts $G_{1,2}$ is:

$$\Pi_Z^{G_1 G_2} = \frac{\alpha}{4\pi(c_W s_W)^2} \frac{1}{1+\tau} [\tau(A(\tau) - A(-1)) + (\tau+1)A'(-1)], \qquad (59)$$

with explicit form of $A(\tau)$ given in Appendix D.

The polarization operator for W-boson has contributions from the loop Feynman diagrams with (ν, e), $(\bar{d} + \bar{s})(u + c)$, (W, Z), (W, γ) and the states with ghosts. Defining the dimensionless combination:

$$\Pi^W = -\frac{i}{q^2 - m_W^2} \Pi_{\text{tr}}^Z(q^2) = \Pi^W(\tau_W), \qquad \tau_W = -\frac{q^2}{m_W^2}, \qquad (60)$$

we write

$$\Pi^W = \Pi_W^{l\bar{\nu}_l} + \sum_q \Pi_W^q + \Pi_W^{WZ} + \Pi_W^{WG_Z} + \Pi_W^{G_W, G_Z} + \Pi_W^{W,\gamma} + \Pi_W^{G_W,\gamma}. \qquad (61)$$

24

From now on, when considering the definite contributions to Π^W, we imply that $\tau \to \tau_W$. Let us first consider the contributions from fermions. For the state with a charged lepton and the corresponding antineutrino we obtain:

$$\sum \Pi_W^{l\bar{\nu}_l} = 3\frac{\alpha}{24\pi s_W^2}\frac{1}{1+\tau}F(\tau),\tag{62}$$

with function F given in (57). Factor 3 corresponds to the number of lepton generations. The contribution of quark states is:

$$\sum_{q=u,d,s,c} \Pi_W^q = 4\frac{\alpha}{24\pi s_W^2}\frac{1}{1+\tau}F(\tau),\tag{63}$$

where factor 4 corresponds to the number of pairs $(\bar{d}+\bar{s})(u+c)$. The for the WZ state we have:

$$\Pi_W^{WZ} = -\frac{\alpha c_W^2}{4s_W^2(1+\tau_W)}[\Psi(\tau_W) - \Psi(-1) - (1+\tau_W)\Psi'(-1)],\tag{64}$$

$$\Psi(z) = \left(4z - 1 - \frac{1}{c_W^2}\right)\int_0^1 \log\left(x + \frac{1-x}{c_W^2} + x(1-x)z\right)dx -$$

$$- \left(\frac{1}{12}z + \frac{1}{3}\right)\int_0^1 \log(1 + x(1-x)z)dx +$$

$$+ \frac{1}{2}s_W^2\int_0^1 dx\int_0^1 y\log\left(y + (1-y)\left(x + \frac{1-x}{c_W^2}\right) + y(1-y)z\right)dy,$$

$$\Psi(-1) = 0.226, \qquad \Psi'(-1) = -1.26.$$

Now we consider the intermediate states (W, G_Z) and (G_W, G_Z). For the

insertion to the box amplitude we have:

$$\Pi_W^{WG_Z} + \Pi_W^{G_W, G_Z} =$$

$$= -\frac{\alpha}{2\pi} \frac{1}{1+\tau} \left[\int_0^1 dx \log \left(\frac{x + (1-x)(1+\tau x)c_W^2}{x + (1-x)^2 c_W^2} \right) - \right.$$

$$\left. - \tau \left(A(\tau) - A\left(-\frac{1}{c_W^2} \right) \right) - (\tau + c_W^2) A'\left(-\frac{1}{c_W^2} \right) \right], \qquad (65)$$

with $A(\tau)$ taken with $\gamma = 1/c_W^2$.

For the last two terms we have:

$$\Pi_W^{W,\gamma} + \Pi_W^{G_W,\gamma} = \frac{\alpha}{4\pi(\tau+1)} \left(-4Q(\tau) + \frac{5}{36} R(\tau) \right),$$

$$Q(\tau) = \tau \int_0^1 dx \log(1 + \tau x) + 3 + (1+\tau)\left(1 + \frac{1}{2} \log \frac{m_Z^2}{\lambda^2} \right).$$

$$R(\tau) = -\frac{6}{\tau^2} - \frac{15}{\tau} + 11 + 6\left(\frac{1+\tau}{\tau} \right)^3 \log(1+\tau) - 20 - 27(1+\tau),$$

Note that the term $\log\left(m^2/\lambda^2\right)$ in the expression for $Q(\tau)$ is compensated by the corresponding contributions from the two-box amplitudes.

Let us now consider the contributions to the transition polarization $\Pi_{\mu\nu}^{Z\gamma} = \Pi^{Z\gamma} g_{\mu\nu}$, and define the dimensionless function

$$\Pi^{Z\gamma} = -\frac{i}{q^2} \Pi_{\text{tr}}^{Z\gamma}(q^2). \qquad (66)$$

As shown above, the fermions contribution is proportional to a^2 and can be omitted. The contributions of $(W^+ W^-)$, $(W^\pm G_W^\mp)$, $(G_W^\pm G_W^\mp)$ to $\Pi^{Z\gamma}$ are, respectively:

$$-i\frac{\alpha c_W}{8\pi s_W}\left(-\frac{19}{6} + \frac{16}{3\tau_W} \right) L(\tau_W), \qquad -i\frac{\alpha c_W}{8\pi s_W}\left(\frac{1}{6} + \frac{2}{3\tau_W} \right) L(\tau_W),$$

$$i\frac{\alpha c_W^3}{2\pi s_W \tau_W} L(\tau_W).$$

26

Thus, the total is:

$$\Pi^{Z\gamma} = -\frac{i\alpha c_W}{8\pi s_W}\left(-3 + \frac{1}{\tau_W}(6 - 4c_W^2)\right)L(\tau_W). \tag{67}$$

D   Loop integrals and regularization

To calculate loop integrals, we perform the Wick rotation of the loop momentum k ($k_0 \to ik_4$, $k^2 = -k_E^2 < 0$). In order to regularize ultra-violet divergence, we introduce the cut-off parameter Λ so $k_E^2 < \Lambda^2$, and all of the kinematical invariants much less (i.e. $\Lambda^2 \gg |p_i p_j|$). The final result will be independent of Λ after the renormalization procedure. Let us now list all the integrals we need:

$$\begin{aligned}
\int \frac{k^2 dk}{(k^2 - D)^3} &= \log\frac{\Lambda^2}{D} - \frac{3}{2}, & \int \frac{dk}{(k^2 - D)^2} &= \log\frac{\Lambda^2}{D} - 1, \\
\int \frac{dk}{(k^2 - D)^3} &= -\frac{1}{2D}, & \int \frac{dk}{(k^2 - D)^4} &= \frac{1}{6D^2}, \\
\int \frac{(k^2)^2 dk}{(k^2 - D)^4} &= \log\frac{\Lambda^2}{D} - \frac{11}{6}, & \int \frac{k^2 dk}{(k^2 - D)^4} &= -\frac{1}{3D}.
\end{aligned} \tag{68}$$

Here, we use the notation $dk \equiv d^4 k/(i\pi^2) = k_E^2 dk_E^2$, where k_E is the Euclidean 4-vector (i.e. $k_E^2 = k_1^2 + k_2^2 + k_3^2 + k_4^2 > 0$) and omit the terms of order $O(D/\Lambda^2)$. We also use the consequence of the integrand symmetry:

$$\int f\left(k^2\right) k_\mu dk = 0 \tag{69}$$

for any function $f(k^2)$. The standard procedure of shifting variable in loop integrals [32] leads to:

$$\int \frac{dk}{((k-b)^2 - d)^2} = \log\frac{\Lambda^2}{d} - 1, \quad \int \frac{k_\mu dk}{((k-b)^2 - d)^2} = b_\mu \left(\log\frac{\Lambda^2}{d} - \frac{3}{2}\right).$$

27

Let us consider the divergent integrals with $A \equiv k^2 - m^2$ and $B \equiv (q-k)^2 - m^2$:

$$\int \frac{dk}{AB} = L_\Lambda - 1 - L,$$

$$\int \frac{k_\mu dk}{AB} = \frac{1}{2} q_\mu \left(L_\Lambda - \frac{3}{2} - L \right),$$

$$\int \frac{k_\mu k_\nu dk}{AB} = g_{\mu\nu} \left\{ -\frac{\Lambda^2}{4} + \frac{q^2}{72} - \frac{m^2}{4} + \frac{1}{2} \left(m^2 - \frac{q^2}{6} \right) L_\Lambda + \frac{1}{3} \left(\frac{q^2}{4} - m^2 \right) L \right\} +$$

$$+ q_\mu q_\nu \left\{ \frac{1}{3} L_\Lambda - \frac{5}{9} + \frac{1}{3} \left(m^2 - q^2 \right) L \right\}, \tag{70}$$

where

$$L_\Lambda = \log \frac{\Lambda^2}{m^2}, \qquad L = L(\tau) = \int\limits_0^1 dx \log(1 + x(1-x)\tau), \qquad \tau = -\frac{q^2}{m^2}.$$

By contracting indices in the tensor integral (70), we obtain:

$$\int \frac{k^2 dk}{AB} = -\Lambda^2 - \frac{q^2}{2} - m^2 + 2m^2 L_\Lambda - m^2 L. \tag{71}$$

According the renormalization procedure, we can omit terms having the form $aq^2 + bm^2$ and $(cq^2 + dm^2)L_\lambda$.

Let us consider now the general integral of the form

$$I_{\mu\nu} = \int \frac{k_\mu k_\nu dk}{(k^2 - m_1^2)((k-q)^2 - m_2^2)}. \tag{72}$$

Now, let us use the following algebraic identity:

$$\frac{1}{(q-k)^2 - m_2^2} = \frac{1}{k^2 - m_2^2} + \frac{2qk - q^2}{(k^2 - m_2^2)^2} + \frac{(2qk - q^2)^2}{(k^2 - m_2^2)^2((q-k)^2 - m_2^2)}. \tag{73}$$

Due to our renormalization convention, we can omit the first and the second terms in the right-hand side of this equation so the integral reads as:

$$I_{\mu\nu} = \int \frac{k_\mu k_\nu (2qk - q^2)^2 dk}{(k^2 - m_1^2)(k^2 - m_2^2)^2((k-q)^2 - m_2^2)}. \tag{74}$$

28

First, we combine the factors $(k^2 - m_1^2)$ and $(k^2 - m_2^2)^2$ in the denominator using the Feynman trick:

$$\frac{1}{a^2 b} = 2 \int\limits_0^1 \frac{(1-x)dx}{(a(1-x) + bx)^3} \tag{75}$$

and obtain

$$\frac{1}{(k^2 - m_1^2)(k^2 - m_2^2)^2} = 2 \int\limits_0^1 \frac{(1-x)dx}{(k^2 - M_x^2)^3}, \qquad M_x^2 = (1-x)m_2^2 + xm_1^2.$$

Next, we join the resulting expression with the factor $((k-q)^2 - m_2^2)$ with the similar Feynman identity:

$$\frac{1}{c^3 d} = 3 \int\limits_0^1 \frac{(1-y)^3 dy}{(c(1-y) + dy)^4}. \tag{76}$$

and, finally, get:

$$\frac{1}{(k^2 - M_x^2)^3((q-k)^2 - m_2^2)} = 3 \int\limits_0^1 \frac{(1-y)^2 dy}{((k - yq)^2 - m_1^2 d)^4}, \tag{77}$$

where

$$d = \tau_1 y(1-y) + \mu^2, \qquad \mu^2 = x(1-y) + \gamma[y + (1-x)(1-y)],$$

$$\tau_1 = -\frac{q^2}{m_1^2}, \qquad \gamma = \frac{m_2^2}{m_1^2}.$$

Thus, we have the logarithmically-divergent loop momentum integral, which allows the operation of the loop momentum shifting $k = \bar{k} + qy$. After that, we can use the loop integrals from the beginning of this Appendix. Now, we have:

$$I_{\mu\nu} = A(\tau_1, \gamma) \ q^2 g_{\mu\nu} + O(q_\mu q_\nu),$$

$$A(\tau_1, \gamma) = - \int\limits_0^1 dx \int\limits_0^1 dy(1-x)(1-y)^2 \left(\log d - \frac{\tau_1(1-2y)^2}{2d} \right), \tag{78}$$

29

therefore the renormalization procedure for this integral has the form:

$$\tau_1 A(\tau_1, \gamma) \to \tau_1(A(\tau_1, \gamma) - A(-1, \gamma)) + (1 + \tau_1)A'(-1, \gamma), \qquad (79)$$

where $A(-1, \gamma) \approx -0.0896$ and $A'(-1, \gamma) \approx 0.00654$ for $\gamma = m_H^2/m_Z^2 = 1.879$.

References

[1] *Møller C.* Zur Theorie des Durchgangs schneller Elektronen durch Materie // Ann. der Phys. 1932. V. 406. P. 531.

[2] *Swartz M. et al.* Observation of Target Electron Momentum Effects in Single Arm Moller Polarimetry // Nucl. Instrum. Meth. A. 1995. V. 363. P. 526.

[3] *Steiner P., Feltham A., Sick I., Zeier M., Zihlmann B.* A High-Rate Coincidence Moller Polarimeter // Nucl. Instrum. Meth. A. 1998. V. 419. P. 105.

[4] *Band H., Mitchell G., Prepost R., Wright T.* A Moller Polarimeter for High-Energy Electron Beams // Nucl. Instrum. Meth. A. 1997. V. 400. P. 24.

[5] *Hauger M., Honegger A., Jourdan J., Kubon G., Petitjean T., Rohe D., Sick I., Warren G., Woehrle H., Zhao J., Ent R., Mitchell J., Crabb D., Tobias A., Zeier M., Zihlmann B.* A High Precision Polarimeter // Nucl. Instrum. Meth. A. 2001. V. 462. P. 382.

[6] *Arrington J., Beise E.J., Filippone B.W., O'Neill T.G., Dodge W.R., Dodson G.W., Dow K.A., Zumbro J.D.* A Variable Energy Moller Polarimeter at the MIT Bates Linear Accelerator Center // Nucl. Instrum. Meth. A. 1992. V. 311. P. 39.

[7] *Alexander G., Cohen I.* Moller Scattering Polarimetry for High-Energy e^+e^- Linear Colliders // Nucl. Instrum. Meth. A. 2002. V. 486. P. 552.

[8] *Derman E., Marciano W.J.* Parity Violating Asymmetries in Polarized Electron Scattering // Ann. Phys. 1979. V. 121. P. 147.

[9] *Kumar K.S., Hughes E., Holmes R., Souder P.* Precision Low-Energy Weak Neutral Current Experiments // Mod. Phys. Lett. A. 1995. V. 10. P. 2979.

[10] *Kumar K.* The E158 Experiment // Eur. Phys. J. A. 2007. V. 32. P. 531.

[11] *Anthony P. et al. (SLAC E158 Collaboration)* Observation of Parity Nonconservation in Moller Scattering // Phys. Rev. Lett. 2004. V. 92. P. 181602.

[12] *Anthony P. et al. (SLAC E158 Collaboration)* Precision Measurement of the Weak Mixing Angle in Moller Scattering // Phys. Rev. Lett. 2005. V. 95. P. 081601.

[13] *Androic D. et al. (Qweak Collaboration)* First Determination of the Weak Charge of the Proton // Phys. Rev. Lett. 2013. V. 111. P. 141803.

[14] *van Oers W.T.H. (MOLLER Collaboration)* The MOLLER Experiment at Jefferson Lab: Search for Physics Beyond the Standard Model // AIP Conf. Proc. 2010. V. 1261. P. 179.

[15] *Benesch J.* . http://hallaweb.jlab.org/12GeV/Moller/downloads/DOE_Proposal/DOE_Moller 2011.

[16] *Kumar K. S.* Parity-Violating Moeller Scattering // AIP Conf. Proc. 2009. V. 1182. P. 660.

[17] *J. Benesch . (MOLLER Collaboration)* The MOLLER Experiment: An Ultra-Precise Measurement of the Weak Mixing Angle Using Møller Scattering: arXiv:1411.4088.

[18] *Aleksejevs A.G., Barkanova S.G., Ilyichev A.N., Zykunov V.A.* Electroweak Radiative Corrections for Polarized Moller Scattering at Future 11 GeV JLab Experiment // Phys. Rev. D. 2010. V. 82. P. 093013.

[19] *Aleksejevs A.G., Barkanova S.G., Zykunov V.A.* Precise Calculations of Observables of Polarized Moller Scattering: From JLAB to ILC Energies // Phys. Atom. Nucl. 2012. V. 75. P. 209.

[20] *Aleksejevs A.G., Barkanova S.G., Ilyichev A.N., Kolomensky Yu.G., Zykunov V.A.* One-Loop Electroweak Corrections for Polarized Moller Scattering at Different Renormalization Schemes and Conditions: arXiv:1010.4185.

[21] *Czarnecki A., Marciano W.J.* Electroweak Radiative Corrections to Polarized Moller Scattering Asymmetries // Phys. Rev. D. 1996. V. 53. P. 1066.

[22] *Petriello F.J.* Radiative Corrections to Fixed Target Moller Scattering Including Hard Bremsstrahlung Effects // Phys. Rev. D. 2003. V. 67. P. 033006.

[23] *Aleksejevs A.G., Barkanova S.G., Kolomensky Yu.G., Kuraev E.A., Zykunov V.A.* Quadratic Electroweak Corrections for Polarized Moller Scattering // Phys. Rev. D. 2012. V. 85. P. 013007.

[24] *Aleksejevs A.G., Barkanova S.G., Bystritskiy Yu.M., Kuraev E.A., Ilyichev A.N., Zykunov V.A.* Parity Violating Moller Scattering Asymmetry Up to the Two-Loop Level: arXiv:1202.0378.

[25] *Berends F.A., Kleiss R., De Causmaecker P., Gastmans R., Troost W., Wu T.T.* Multiple Bremsstrahlung in Gauge Theories at High-Energies. 2. Single Bremsstrahlung // Nucl. Phys. B. 1982. V. 206. P. 61.

[26] *Berends F.A., Kleiss R., De Causmaecker P., Gastmans R., Wu T.T.* Single Bremsstrahlung Processes in Gauge Theories // Phys. Lett. B. 1981. V. 103. P. 124.

[27] *Ahmadov A.I., Bystritskiy Yu.M., Kuraev E.A., Ilyichev A.N., Zykunov V.A.* One-Loop Chiral Amplitudes of Moller Scattering Process // Eur. Phys. J. C. 2012. V. 72. P. 1977.

[28] *Yennie D.R., Frautschi S.C., Suura H.* The Infrared Divergence Phenomena and High-Energy Processes // Ann. Phys. 1961. V. 13. P. 379.

[29] *Amsler C. et al. (Particle Data Group Collaboration)* // Phys. Lett. B. 2008. V. 667. P. 1.

[30] *Jegerlehner F.* Hadronic Contributions to the Photon Vacuum Polarization and Their Role in Precision Physics // J. Phys. G. 2003. V. 29. P. 101.

[31] *Karplus R., Kroll N.M.* Fourth-Order Corrections in Quantum Electrodynamics and the Magnetic Moment of the Electron // Phys. Rev. 1950. V. 77. P. 536.

[32] *Akhiezer A.I., Berestetskij V.B.* Quantum Electrodynamics. Moscow: Nauka, 1981. 428 p.

[33] *Aleksejevs A.G., Barkanova S.G., Bystritskiy Yu.M., Ilyichev A.N., Kuraev E.A., Zykunov V.A.* Double-Box Contributions to Moeller Scattering in the Standard Model // Eur. Phys. J. C. 2012. V. 72. P. 2249.

[34] *Cheng T.-P., Li L.-F.* Gauge Theory of Elementary Particle Physics. USA: Oxford University Press, 1988. 624 p.

[35] *Denner A.* Techniques for Calculation of Electroweak Radiative Corrections at the One Loop Level and Results for W Physics at LEP-200 // Fortschr. Phys. 1993. V. 41. P. 307.